

# Mathematics for Management

An Introduction

M Raghavachari





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# GLOSSARY OF GREEK ALPHABETS

$\alpha$	alpha	$\beta$	beta
$\gamma, \Gamma$	gamma	$\delta, \Delta$	delta
$\epsilon$	epsilon	$\nu$	nu
$\eta$	eta	$\theta$	theta
$\lambda, \Lambda$	lambda	$\mu$	mu
$\rho$	rho	$\pi$	pi
$\tau$	tau	$\sigma, \Sigma$	sigma
$\chi$	chi	$\phi, \Phi$	phi

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